True-False p. 266

1. True (sum rule)
   "The derivative of the sum is the sum of the derivatives."

2. FALSE (incorrect version of product rule)
   The derivative of the product IS NOT the product of the derivatives.
   Instead: $\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + g'(x)f(x)$

3. True (Chain Rule)

4. True (Chain Rule for square root)
   Notice that
   $$\frac{d}{dx}(\sqrt{x}) = \frac{d}{dx}(x^{\frac{1}{2}}) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$
   $$\xrightarrow[\text{Rule}]{\text{Chain}} \frac{d}{dx}(\sqrt{f(x)}) = \frac{1}{2\sqrt{f(x)}}f'(x) = \frac{f'(x)}{2\sqrt{f(x)}}$$

5. FALSE Compare this to 4 ↑
   In this case, f is the "outer" function and $\sqrt{x}$ is the "inner" function, so
   $$\frac{d}{dx}(f(\sqrt{x})) = f'(\sqrt{x})\cdot\frac{d}{dx}(\sqrt{x}) = f'(\sqrt{x})\cdot\frac{1}{2\sqrt{x}}$$
   ↑ Not $f'(x)$ ↑

6. False e^2 is a constant, so $\frac{d}{dx}(e^2) = 0$.

7. FALSE Remember, the power rule applies when the base is x, like

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

but the exponential rule applies when x is in the exponent:

$$\frac{d}{dx}(10^x) = \ln 10 \cdot 10^x$$

8. False $\ln 10 \approx 2.302585$, a constant, so $\frac{d}{dx}(\ln 10) = 0$. (There is no x!)

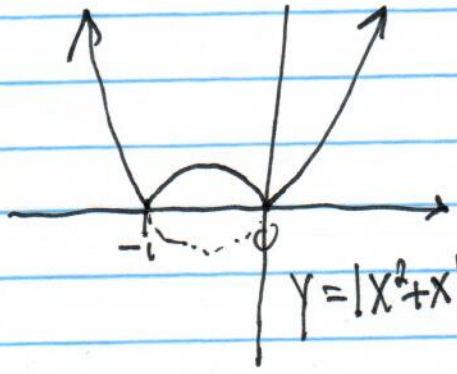
9. True $\frac{d}{dx}(\tan^2 x) = 2(\tan x)' \cdot \sec^2 x$ ↔ Equal!!

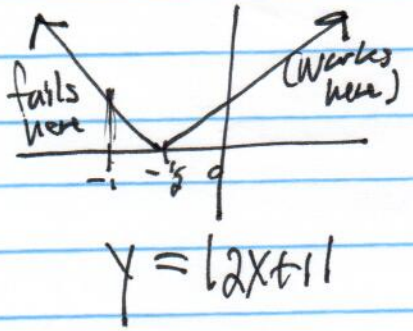
By chain Rule: $\frac{d}{dx}(\sec^2 x) = 2(\sec x)' \cdot \sec x \tan x$

You might also recall $\sec^2 x = \tan^2 x + 1$. Whenever two functions differ by a constant, their derivatives are equal.

10. False In general, $\frac{d}{dx}|f(x)| \neq |f'(x)|$

Notice in particular that if $x < -1$, $|x^2+x| = x^2+x$, and the slope is negative, but $|2x+1| > 0$.





11. True — True by power rule. (Note that it may be a degree zero "polynomial," like $f(x)=10$ or $f(x)=0$.)

12. True — Without actually multiplying out $(x^6-x^4)^5$ we can see the degree is $6\cdot5=30$, so the 31st derivative is 0.

13. True — True by the quotient rule. (See page 30 for the definition of a rational function, which technically includes polynomials is the same way $7=\frac{7}{1}$ is a rational number.)

14. FALSE! — Notice that $y-4=2x(x+2)$ is equivalent to

$$y-4=2x^2+4x$$

which is a quadratic, ~~not~~ a line. But this is a common mistake: after you find ~~slope~~ $f'(x)=2x$, you need to put in $x=2$ to get $m=2\cdot2=4$.

15. True — We can evaluate $\lim_{x\to 2}\frac{g(x)-g(2)}{x-2}$ directly if we want, but we don't need to since we see that is $g'(2)$. & $g'(x)=5x^4$ so $g'(2)=5\cdot2^4=80$.